House prices example

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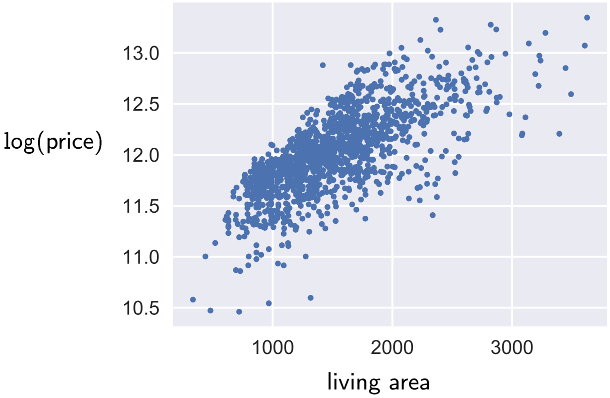
The data set

- sale prices of $n = 1456$ homes in Ames, Iowa from 2006 to 2010
- goal is to predict $\log(\text{price})$
- performance metric is RMS error on test set
- *e.g.*, RMS error of 0.1 means (roughly) we can predict house price within factor $e^{0.1}$ (about 10.5%)

Scatter plot of price versus living area



Embedding

- $v =$ price, let $y = \log(v)$
- 9 numerical fields are embedded unchanged
  - *year built, area of living space, area of first floor, area of second floor, area of garage, area of wooden deck, area of basement, year of last remodel, area of lot*
- 8 ordinal fields are embedded as integers
  - *number of bedrooms, number of kitchens, number of fireplaces, number of half bathrooms, number of rooms, condition (scored 1-10), quality of materials and finish (scored 1-10), car capacity of garage*

Embedding

- *kitchen quality:* on likert scale

  EXCELLENT, GOOD, TYPICAL, FAIR

  embedded as integer between 1 and 5

- *building type:* 5 categories, one-hot embedded

  SINGLE-FAMILY TOWNHOUSE END UNIT TWO-FAMILY-CONVERSION

  TOWNHOUSE INSIDE UNIT DUPLEX

- *neighborhood:* 25 categories, one-hot embedded
- results in matrix $X_0 \in \mathbf{R}^{n \times 48}$

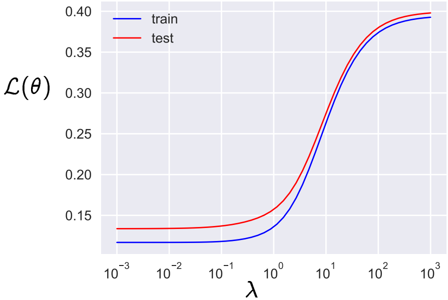
Standardization and data splitting

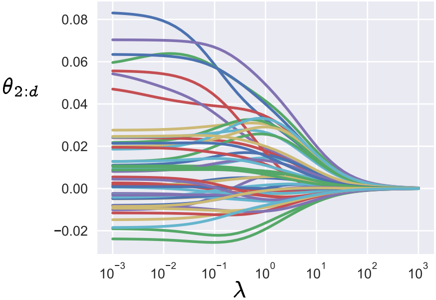
- split randomly 80/20 into training and test sets
- gives $X_0^{\text{train}} \in \mathbf{R}^{1165\times 48}$, $Y^{\text{train}} \in \mathbf{R}^{1165}$ and $X_0^{\text{test}} \in \mathbf{R}^{291\times 48}$ and $Y^{\text{test}} \in \mathbf{R}^{291}$
- use *training set* to compute means and standard deviations of each column of X_0^{train}
- use means and stds to standardize X_0^{train} and X_0^{test}
  - both datasets are standardized using the *same* means/stds (from the training set)
  - some columns of X_0^{train} may have zero standard deviation (e.g. categoricals which occur rarely); special case since standardization formula does not apply
- append a constant feature; let $X^{\text{train}} = \begin{bmatrix} \mathbf{1} & \text{standardize}(X_0^{\text{train}}) \end{bmatrix}$ and $X^{\text{test}} = \begin{bmatrix} \mathbf{1} & \text{standardize}(X_0^{\text{test}}) \end{bmatrix}$

Ridge regression

- choose a range of λ values, logarithmically spaced between 10^{-3} and 10^{3}
- for each λ, compute
  - RERM: the θ that minimizes $\frac{1}{n}||X^{\text{train}}\theta - Y^{\text{train}}||^2 + \lambda||\theta_{2:d}||^2$
  - the training error $\text{rms}(X^{\text{train}}\theta - Y^{\text{train}})$
  - the test error $\text{rms}(X^{\text{test}}\theta - Y^{\text{test}})$

Ridge regression

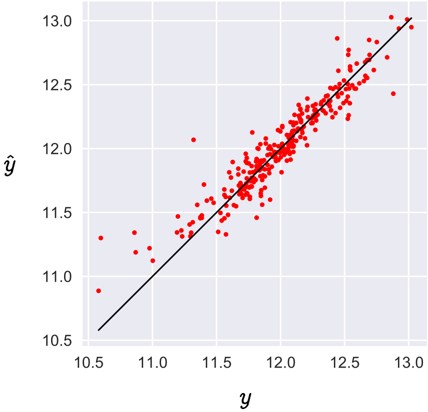




- no benefit of regularization in this case
- minimum rms error on test set is about 0.12
- corresponds to about 13% error in house price

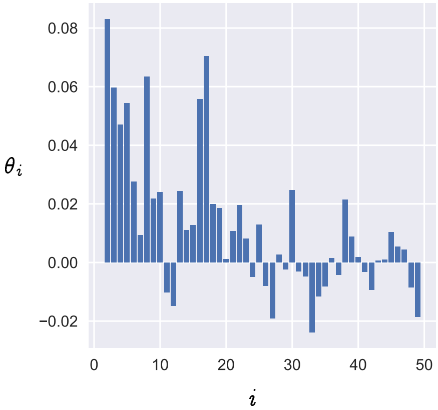
Results

- plot shows all test points



Important features

- θ_2: year built
- θ_3: area of living space
- θ_4: area of first floor
- θ_5: area of second floor
- θ_8: area of basement
- θ_{16}: condition
- θ_{17}: quality of materials and finish
- difference between best and worst neighborhoods is 4% price



Julia

```
D, header = loaddata()
n = size(D,1)
Y = embedy(D, header)
X0 = embedx(D, header)
trainrows, testrows = randomsplit(n)
Xtrain0, Ytrain, Xtest0, Ytest = applysplit(X0, Y, trainrows, testrows)
means, stds = getstatistics(Xtrain0)
Xtrain = standardizeplusone(Xtrain0, means, stds)
Xtest  = standardizeplusone(Xtest0, means, stds)
lambdas = 10 .^ range(-3,3,length=50)
thetas = [ridgeregressionconstfeature(Xtrain, Ytrain, lambda) for lambda in lambdas]
train_errors = [rmse(Xtrain*theta, Ytrain) for theta in thetas]
test_errors  = [rmse(Xtest*theta, Ytest) for theta in thetas]
```

Julia

```
function randomsplit(n, trainfrac=0.8)
    ntrain = convert(Int64, round(trainfrac*n))
    p = Random.randperm(n)
    trainrows = sort(p[1:ntrain])
    testrows = sort(p[ntrain+1:n])
    return trainrows, testrows
end

function applysplit(X, Y, trainrows, testrows)
    return X[trainrows,:], Y[trainrows,:], X[testrows,:],  Y[testrows,:]
end
```

Julia

```
function getstatistics(U)
    means = [Statistics.mean(x) for x in eachcol(U)]
    stds  = [Statistics.std(x) for x in eachcol(U)]
    return means, stds
end
```

Julia

```
function standardizeplusone(X,means,stds)
    Z = zeros(size(X))
    for i=1:size(X,2)
        if stds[i] != 0
            Z[:,i] = (X[:,i] .- means[i])/stds[i]
        else
            Z[:,i] = X[:,i] .- means[i]
        end
    end
    n = size(X,1)
    Z = [ones(n,1) Z]
    return Z
end
```

Julia

```
function embedx(D, header)
    field(name) = getdatafield(D, header, name)
    realf(name) = stringtonumber.(field(name))
    X = hcat(realf("YearBuilt"),    # numeric
             realf("GrLivArea"),    # numeric
             realf("1stFlrSF"),     # numeric
              ...
             realf("GarageCars"),   # ordinal 0-4
             unlikert.(field("KitchenQual")),  # ordinal, but "Ex", "Gd", "TA", "Fa", "Po"
             onehot(field("Neighborhood")),   # 25 different names
             onehot(field("BldgType")),       # 5 different types
            )
    return X
end
```

Julia

```
function unlikert(s)
    d = Dict("Ex" =>5, "Gd" =>4, "TA" => 3, "Fa" => 2, "Po" => 1)
    return d[s]
end


embedy(D, header) = log.(stringtonumber.(getdatafield(D, header, "SalePrice")))
```

Julia

```
# takes a list length n, e.g. u = ["hi", "lo", "hi", "med", "lo"]
# returns a matrix Y which is n by d
function onehot(u)
    categories = unique(u)
    catnum(s) = findfirst(x -> x==s, categories)
    n = length(u)
    K = length(categories)
    Y = zeros(n,K)
    for i=1:n
        c = catnum(u[i])
        Y[i,c] = 1
    end
    return Y
end
```

Julia

```
function ridgeregressionconstfeature(X,Y,lambda)
    n,d = size(X)
    m = size(Y,2)
    E = [zeros(d-1,1)  I(d-1)]
    A = [X; sqrt(lambda*n)*E]
    B = [Y; zeros(d-1,m)]
    theta = A\B
end
```